**Specialist Mathematics Units 3 & 4**

**Test 3 2018**

**Calculator Free Section**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Score: \_\_\_\_\_ / 22**

**Time Allowed**: 20 minutes

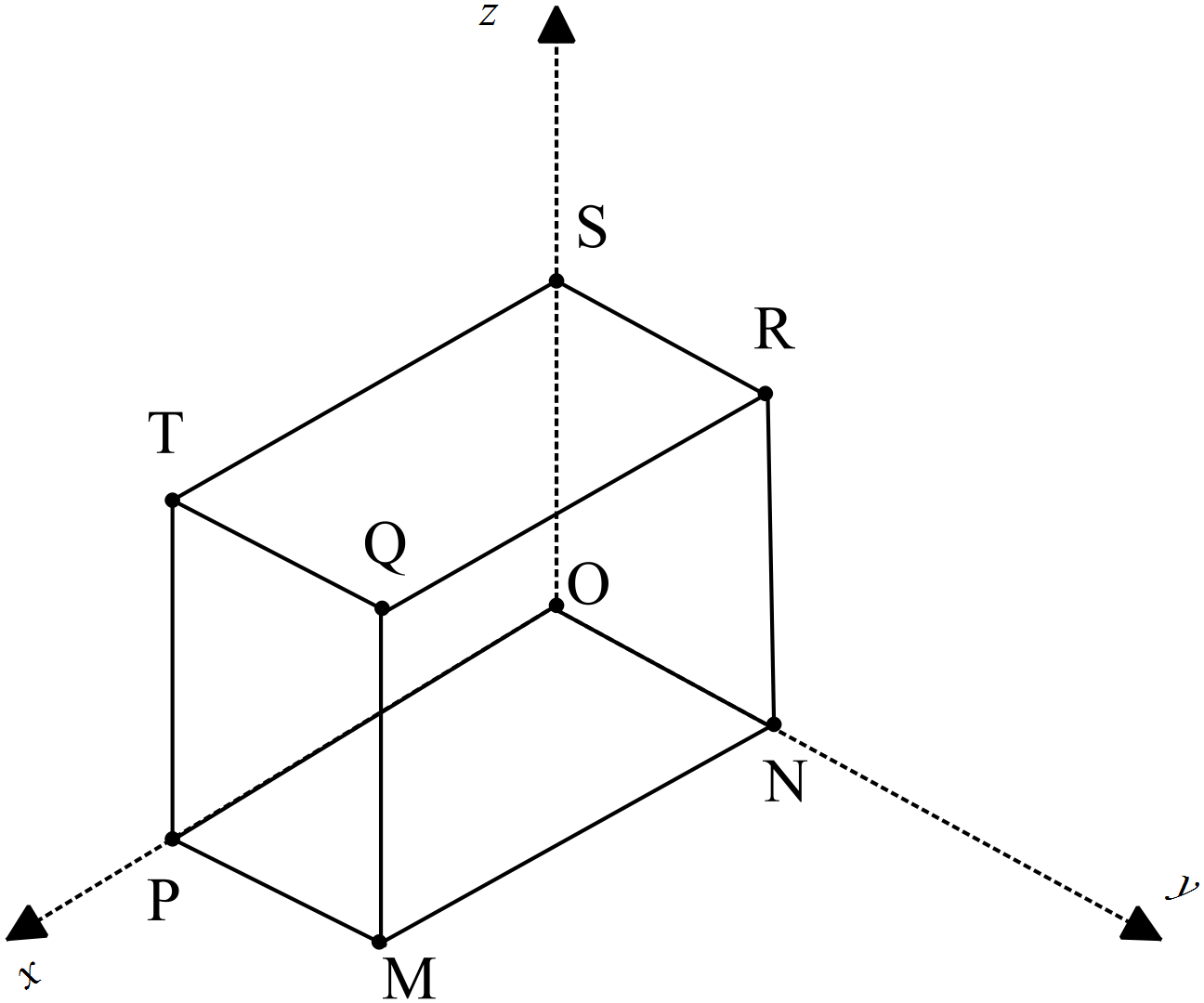
**No calculators or notes are to be used.**

**Access to approved Mathematics Specialist formulae sheet is permitted.**

Where a question or part of a question is worth more than 2 marks sufficient working to justify your solution is required

**1. [ 4 marks ]**

MNOPQRST is a rectangular prism with **OP = *p***, **ON = *n* and OS = *s*.**



**O** is the origin of the ***x***-***y***-***z*** axes.

Use the fact that and **to** prove that the volume of the cuboid is

The area of the base is because it equals since **p** and **n** are perpendicular and =1 ✓

The height is , therefore the volume is ✓

**p**×**n and are parallel so**

or = ✓

**Therefore** volume of the cuboid ✓

**2. [ 5,1 = 6 marks]**

A plane Γ contains the points P at position **k –** 2**j,** Q at 4**i** + **j** +3**k** and R at 2**k** – **i.**

**a)** Determine a vector equation which contains the normal to the plane.

PR = =

PQ = = or QR = = ✓

PR

=

= ✓✓ (-1 each mistake)

Using P,  **r.n =**  = -23 ✓

So **r.**  ✓

**b)** Express this as a Cartesian equation.

*x* + 6*y -* 11*z* = -23 or *x* + 6*y -* 11*z* + 23 = 0 ✓

**3. [1, 1, 3 = 5 marks ]**

A point P moves in a path according to the parametric equations: where

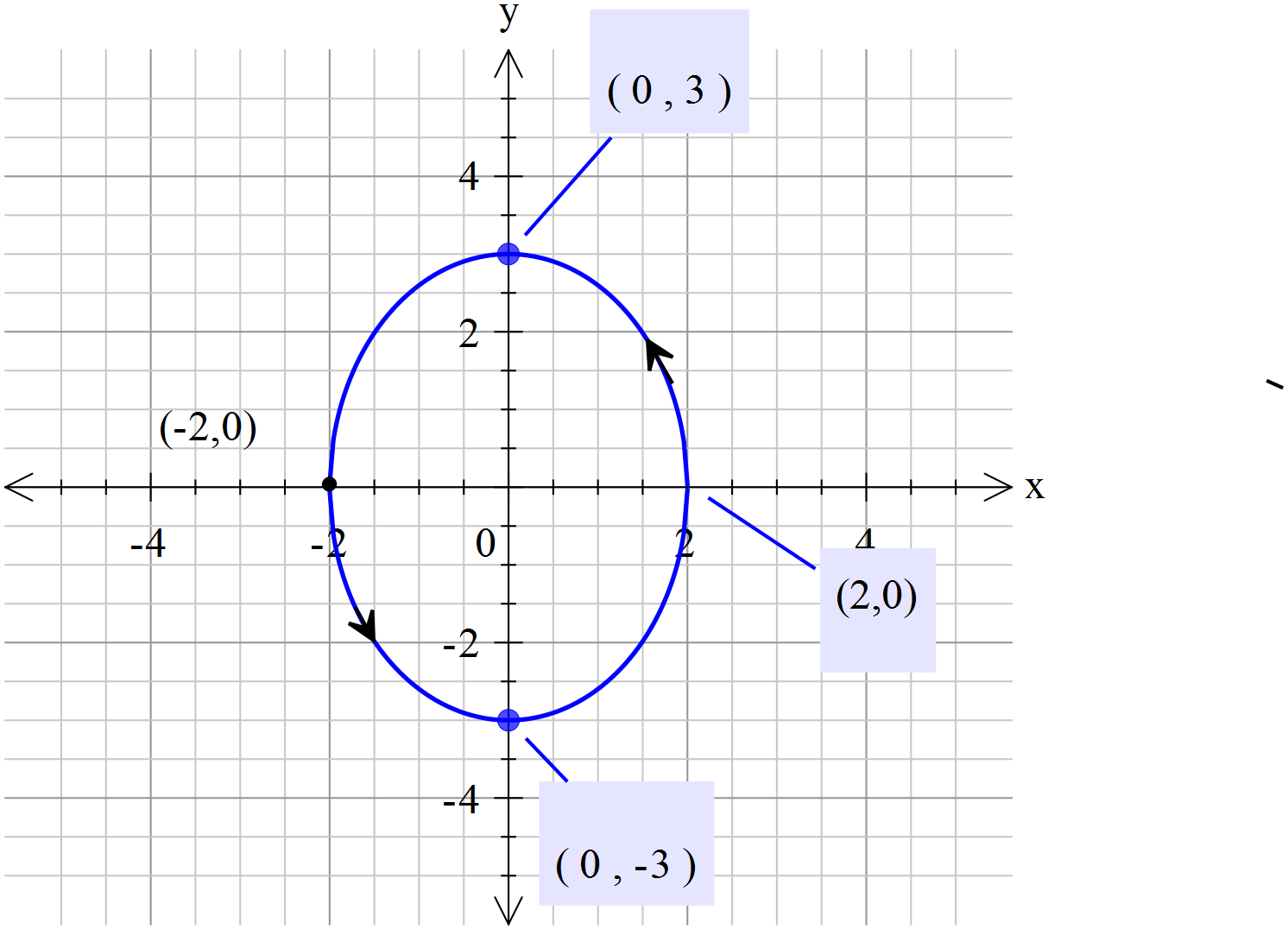
1. Give the Cartesian equation of the path

So ✓

1. State the velocity vector of the ellipse

✓

1. Draw the path traced by P, indicating its direction clearly and key points



Shape of path ✓

Direction correct ✓

Scale/points shown ✓

**4. [4, 1, 2 = 7 marks]**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | + | 3y | - | z | = | 5 |
| 2x | + | 7y | - | 3z | = | 6 |
| -x | - | y | + | (p2-p-1)z | = | p-13 |

For what value of p will the above system of equations have

1. an infinite number of solutions,



 ✓✓

 ✓

An infinite number of solutions when p = 0 ✓

1. no solutions

No solution when p = 1 ✓

1. a unique solution?

A unique solution when pand p  ✓✓

**Specialist Mathematics Units 3 & 4**

**Test 3 2018**

**Calculator Assumed Section**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Score: \_\_\_\_\_ / 25**

**Time Allowed**: 25 minutes

**Materials Allowed**: SCSA formula booklet, SCSA approved calculators and one A4 page of notes (both sides).

**Instructions:** Where a question or part of a question is worth more than 2 marks sufficient working to justify your solution is required.

5. [1,1,3 ,4 = 9 marks ]

The Cartesian equation of plane ∏ is .

**a)** State a vector equation of this plane.



A correct vector equation (many are possible but only one is obvious) ✓

The parametric equations of line  are .

**b)** State a vector equation for line .



A correct vector equation (many are possible but only one is obvious) ✓

**c)** Determine the position vector of , the point of intersection of line  and the plane.



Substitutes expression for **r** in equation of line into equation of plane. ✓

Solves for =-4 ✓

Correct position vector for A ✓

d) Determine the angle between Line L and plane ∏

cos Θ =

**d)** Determine the acute angle between Line L and plane ∏ to the nearest tenth of a degree.

cos Θ =   
  
 = 180 -

= 99.6 , acute version is 80.4

so angle at plane is 9.6 . Alternatively or = = 9.6 .

Uses dot product formula to give an expression for the supplementary angle. ✓

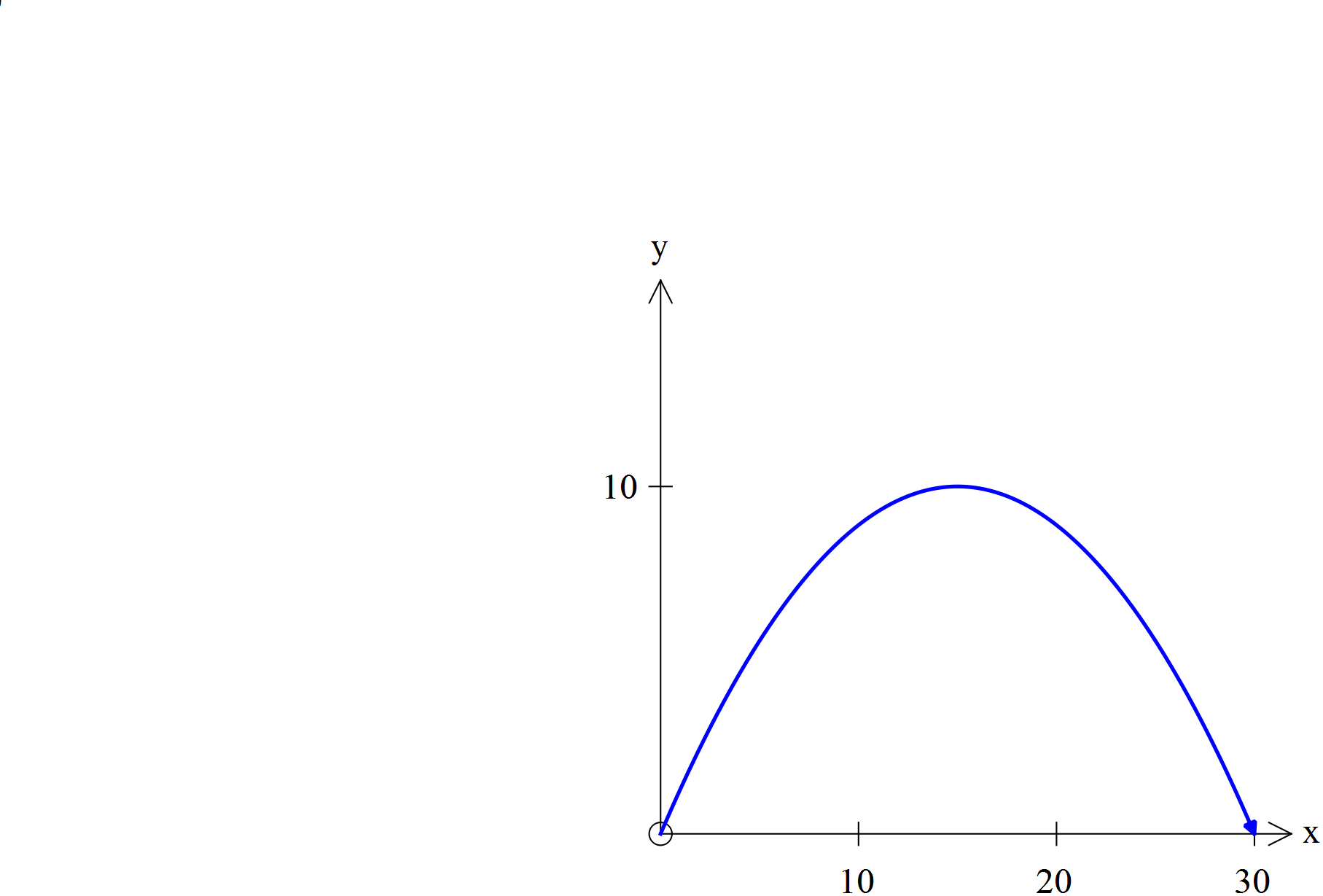
Uses inverse of trig ratio to find angle ✓

Adjusts for acute angle (or uses sin of dot product ) ✓

Correct angle for ✓

6. [ 5, 2, 2, 3, 2 = 12 marks]

For a particle projected from the Origin with speed µ ms-1 at an angle of with the positive x-axis, the position vector at time seconds is metres. The equation of the path of the particle is given by



The diagram featured shows the path of the particle.

The equation of the path of P is .

**a)** State the position vector of P at time seconds.

Comparing the two equations:

So ✓ also sin= and cos=

Also [(5(1+(/] ✓

Therefore ✓ , using calculator

So

✓✓

**b)** Use vector methods to find the direction of the particle at time seconds .

✓

✓

= 21.33185286 or 0.3723110679 radians ✓

**c)** Use vector methods to find the distance travelled by P along its path, from the time of the projection to the time it hits the -axis again. Show clearly how you obtained your answer.

When it hits the ground ✓

Therefore seconds as ✓

Distance along path is ✓

= 37.35938825 metres ✓

**7. [ 4 marks ]**

Prove that the line ***r*** = is a tangent to the sphere with equation (x-2)2+(y-1)2+(z+1)2 =

Change sphere formula to vector equation ✓

Substitute into

Giving ✓

Solving

Gives ✓

Since there is only one point where the line touches the sphere it is a tangent. ✓

**8.** **[ 2,2,2,2= 8 marks ]**

Three particles A, B & C are such that their initial positions relative to the Origin at a given time are < 0, -1, -2> m ; < 4, 1, 6 > m and < -4, 1, -2> m respectively. Their constant velocities are < 1, 2, 2 >ms-1, < -1, α, -2 > ms-1 and < 3, 1, β >ms-1 , also respectively.

**a)** State the position vectors of each particle after *t*  seconds

OA = +t OB = +t OC = +t ✓✓ (-1 each mistake)

or

OA = OB = OC =

**b)** If these velocities are maintained all three will collide. Use vector methods to determine the values of α and β.

Comparing **i**  components , *t = 4-t = 3t-4 giving t = 2* ✓

Comparing **j**  components , 2t+1 = +1 = t + 1

When t = 2, α = 1

Comparing k components , 2t -2 = 6 – 2t =

When t = 2, = 2 ✓

**c)** If these velocities are maintained only A and B will collide. Use vector methods to determine the possible values of α and β in this instance.

A, B & C collide at t = 2 seconds for α = 1 and = 2

For A & B to collide t = 2 and α = 1 ✓

For C not to collide with them ✓

**d)** If these velocities are maintained only two of the three will collide. Use vector methods to determine the possible values of α and β in this instance.

For two only to collide α = 1 or = 2, since when both are true all three collide. If neither are true ie α 1 and 2 , no collision will occur ✓✓